



## INTRODUCTION

Mathematical models can be used to answer questions, solve problems, and make predictions about all kinds of populations. In this activity, you'll use the exponential and logistic growth models in the [Population Dynamics](#) Click & Learn to investigate three different populations of African animals. The models and analyses you'll use here can be used for many other types of populations as well.

### PART 1: Waterbuck

Africa is home to many different kinds of animals, including large antelope called **waterbuck** that live near lakes and rivers. In certain areas, waterbuck populations are declining due to hunting and habitat loss.

1. How could we use mathematical models to help waterbuck and other wildlife?

Let's investigate the waterbuck population in Gorongosa National Park, Mozambique. In the 1970s and 1980s, Mozambique experienced an intense civil war, and most of the waterbuck were killed to provide food and money for soldiers. After the war ended in 1992, many people worked together to rebuild the park. Scientists developed mathematical models to better understand how the park's waterbuck population recovered afterward, and to help make decisions about managing this population in the future.

2. What are the advantages of using a mathematical model to study a population rather than just observing the population?

An early model of the waterbuck population was based on the **exponential growth model**, which is described in the "Exponential growth model" section of the [Population Dynamics](#) Click & Learn. The population's maximum per capita growth rate ( $r$ ) was estimated as the difference between its **per capita birth rate** ( $b$ ), the number of births per individual per unit time, and its **per capita death rate** ( $d$ ), the number of deaths per individual per unit time:

$$r = b - d$$

3. At the start of the recovery period, the waterbuck population contained only 140 individuals. The population had 0.67 births per year per individual and 0.06 deaths per year per individual.
  - a. What is the maximum per capita growth rate ( $r$ ) for this population? Include units in your answer.
  - b. What is the initial population size ( $N_0$ ) for this population? Include units in your answer.

Go to the “**Simulator**” section under the “Exponential growth model” tab in the [Population Dynamics](#) Click & Learn. Fill in the simulator settings based on your answers above. (*Note: The simulator doesn’t show units for times or rates because many units are possible. In these examples, we’ll use “years” as our unit for time and “per year” as our units for per capita rates.*)

4. Using the simulator, fill in the following table with the population size ( $N$ ) and population growth rate ( $dN/dt$ ) at different time points ( $t$ , measured in years).

|                                    |   |    |    |    |    |
|------------------------------------|---|----|----|----|----|
| Time ( $t$ )                       | 5 | 10 | 15 | 20 | 25 |
| Population size ( $N$ )            |   |    |    |    |    |
| Population growth rate ( $dN/dt$ ) |   |    |    |    |    |

5. Based on this model, how will the waterbuck population grow over time? Will the population ever stop growing or get smaller?
6. Do you think this model reflects how the waterbuck population will grow in real life? Why or why not?
7. Imagine that a decrease in the number of predators lowered the per capita death rate of the waterbuck to 0.04 deaths per year per individual.
  - a. What would be the new maximum per capita growth rate ( $r$ ) for the waterbuck population?
  - b. What would be the population size ( $N$ ) after 20 years ( $t = 20$ )? Use the same  $N_0$  as in Question 3.

We originally estimated  $r$  as the difference between the per capita birth rate ( $b$ ) and the per capita death rate ( $d$ ). However,  $r$  is also affected by other processes, such as immigration (movement of individuals *into* a population) and emigration (movement of individuals *out* of a population). Let  $i$  represent the per capita immigration rate and  $m$  represent the per capita emigration rate. The equation for  $r$  can be updated to:

$$r = (b - d) + (i - m)$$

8. Imagine that new waterbuck immigrate into the park at a rate of 0.25 per year. Assume that there are no emigrations and that the rest of the population parameters are the same as in Question 3.
  - a. What would be the population size after 20 years ( $t = 20$ )?

- b. How does the size of the population *with* immigration (your answer to Part A) compare to the size of the population *without* immigration (your result for  $t = 20$  in Table 1)?

## PART 2: Kudu

Another type of African antelope is the kudu. Like waterbuck, many kudu have lost their habitat due to human activities. Male kudu are also hunted for their large spiraled horns, which are taken as trophies. As with waterbuck, developing population models for kudu can help us learn more about them.

Most populations, including those of the waterbuck and kudu, cannot grow forever. They are limited by factors such as food or space, which keep a population from getting too large.

9. Besides food and space, what are **two** other factors that could limit the size of a population?

One model that includes the effect of limiting factors is the **logistic growth model**, which is described in the “Logistic growth model” section of the [Population Dynamics](#) Click & Learn. In this model, a population has an upper limit to its growth called the **carrying capacity** ( $K$ ), which is the largest size of a population that the environment can support in the long run.

Imagine a national park with an initial population of 10 kudu, which have a maximum per capita growth rate of 0.26 per year. The park can support a maximum of 100 kudu in the long run.

10. What are the values of  $K$ ,  $r$ , and  $N_0$  for this kudu population?

Go to the “**Simulator**” section under the “Logistic growth model” tab in the [Population Dynamics](#) Click & Learn. Fill in the simulator settings based on your answers above.

11. Based on this model, about how many years will it take the kudu population to reach the carrying capacity? (*Hint: You may want to change the “Max” values for the axes on Plot 1 to get a better look at the curve.*)
12. What will happen to the population growth rate ( $dN/dt$ ) as the population size ( $N$ ) gets closer and closer to the carrying capacity?
13. Imagine that more land is added to the park, allowing it to support up to 250 kudu. How will the size of the kudu population change once this land is added?
14. Reset the model to the values you determined in Question 10. Now imagine that trophy hunters start killing kudu in the park, which decreases their maximum per capita growth rate to 0.15 per year. How would this impact the kudu’s population size over time? (*Hint: Look at how many years it will take the population to reach its carrying capacity now.*)

### PART 3: Wildebeest

The last type of antelope we'll investigate is the wildebeest, which are found in eastern and southern Africa. Wildebeest live in giant herds that can contain over a million individuals! The wildebeest herd in the Serengeti region of Tanzania is one of the biggest populations of large herbivores in the world.

Before the 1960s, wildebeest and many other hoofed mammals in the Serengeti were killed by rinderpest, a virus related to the measles virus. In 1960, a campaign began to vaccinate domestic cattle, which were a major source of the virus. Over time, the campaign eliminated rinderpest and allowed many animal populations to recover.

Figure 1 shows the population sizes of two animals, wildebeest and zebras, before and after the rinderpest vaccination campaign.

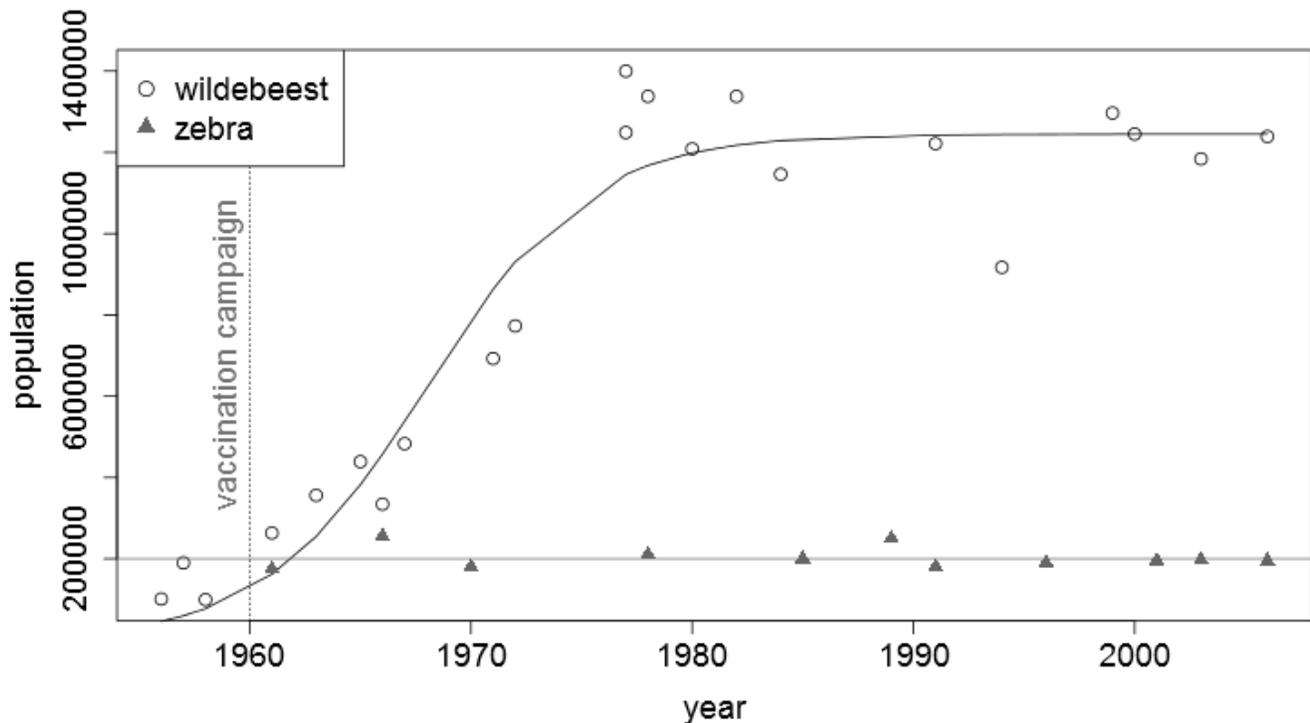


Figure 1. Wildebeest and zebra populations in the Serengeti from the 1950s to 2010.

15. Based on Figure 1, what kind of population growth model would you use to represent the Serengeti wildebeest population? Why?
  
16. Was the wildebeest population at the carrying capacity in 1968? Why or why not?
  
17. Calculate the size of the wildebeest population in the year 1968, using the logistic model simulator with the following settings:  $K = 1,245,000$  wildebeest,  $r = 0.2717$  per year, and  $N_0 = 80,000$  wildebeest in the year 1958.

18. Imagine that the maximum per capita growth rate ( $r$ ) for the wildebeest population increased to 0.4 per year in 1958.
- Suggest a specific reason that  $r$  could *increase* for a population.
  - Recalculate the population size in 1968 using the new  $r$ . You can use the same values for the other settings as in Question 17.
  - Sketch or describe how the wildebeest population curve in Figure 4 might change as a result of the new  $r$ .
19. Imagine that the carrying capacity ( $K$ ) for the wildebeest population decreased to 1,000,000 wildebeest in 1958.
- Suggest a specific reason that  $K$  could *decrease* for a population.
  - Recalculate the population size in 1968 using the new  $K$ . You can use the same values for the other settings as in Question 17.
  - Sketch or describe how the wildebeest population curve in Figure 4 might change as a result of the new  $K$ .
20. Look at the size of the zebra population, which is shown as triangles in Figure 4, before and after the rinderpest vaccination campaign.
- What patterns or trends do you observe in the zebra population?
  - Based on your answer above, what effect does rinderpest have on zebras?
21. Based on Figure 4, did the zebra population growth rate ( $dN/dt$ ) differ in the years 1985 and 2003? Why or why not? (*Hint:  $dN/dt$  is equal to the slope of the curve showing population size,  $N$ , over time,  $t$ .*)
22. Imagine that there is a large wildfire in the Serengeti in 2010.
- How might the zebra and wildebeest populations change right after the wildfire?
  - How large do you think the zebra and wildebeest populations would be 50 years after the wildfire? Explain your answer, or what else you would want to know before making a prediction.

23. We often design population models to answer certain questions. We may leave out other factors that are less relevant to our questions or that could overcomplicate our analysis.
- Propose one *new* question about the waterbuck, kudu, or wildebeest populations that could be answered using the models you learned about in this activity.
  - Propose one *new* question about the waterbuck, kudu, or wildebeest populations that could *not* be answered using these models. What could you add to the models in order to answer your question?